EUF - 2013 2Sem 12/03/2016

Q1. a) Para um fio:

$$E = \frac{\lambda \cdot \cancel{l}}{2\pi r \varepsilon_0 \cancel{l}} = \frac{\lambda}{2\pi r \varepsilon_0}$$

$$r_+ = \sqrt{(x-a)^2 + y^2 + z^2}$$

$$V = \int_a^r \frac{\lambda}{2\pi r \varepsilon_0} = -\frac{\lambda}{2\pi\varepsilon_0} \ln\left(\frac{r_+}{a}\right) = \boxed{\frac{\lambda}{2\pi\varepsilon_0} \ln\left(\frac{\sqrt{(x-a)^2+y^2+z^2}}{a}\right)}$$

b) Considere outro fio em $x = -a$ com $-\lambda$.

$$V_-(x,y,z) = -\frac{(-\lambda)}{2\pi\varepsilon_0} \ln\left(\frac{r_-}{a}\right)$$

$$V_T = V_+ + V_- = \frac{\lambda}{2\pi\varepsilon_0}\left[\ln\left(\frac{r_-}{a}\right) - \ln\left(\frac{r_+}{a}\right)\right] = \frac{\lambda}{2\pi\varepsilon_0} \ln\left(\frac{r_-}{r_+}\right)$$

$$r_- = \sqrt{(x+a)^2 + y^2 + z^2}$$

$$V(x,y,z) = \frac{\lambda}{2\pi\varepsilon_0}\frac{1}{2} \ln\left(\frac{(x+a)^2+y^2+z^2}{(x-a)^2+y^2+z^2}\right)$$

c) surface charge σ induzida

$$\sigma = -\varepsilon_0 \frac{\partial V}{\partial n} \rightarrow \sigma = -\varepsilon_0 \left.\frac{\partial V}{\partial x}\right|_{x=0} = \frac{-\lambda}{4\pi}\left[\frac{2(x+a)}{(x+a)^2+y^2+z^2} - \frac{2(x-a)}{(x-a)^2+y^2+z^2}\right]_{x=0}$$

$$\sigma = -\frac{\lambda}{4\pi}\left[\frac{4a}{a^2+y^2+z^2}\right] = \boxed{-\frac{\lambda a}{\pi(a^2+y^2+z^2)}}$$

$\square$

d) $\displaystyle\int_{-\infty}^{\infty} \frac{\lambda a}{\pi(a^2+y^2+z^2)}\,dy = \quad a_1 = a^2+z^2 \quad = \frac{\lambda a}{\pi}\int_{-\infty}^{\infty}\frac{1}{a_1^2+y^2}\,dy = \left.\frac{\lambda a}{\pi\cdot(a^2+z^2)}\cdot \arctan\left(\frac{y}{a^2+z^2}\right)\right|_{-\infty}^{\infty}$

$$= \frac{\lambda a}{\pi(a^2+z^2)}\cdot\left[\frac{\pi}{2}-\left(-\frac{\pi}{2}\right)\right] = \boxed{\frac{\lambda a}{a^2+z^2}}$$

?

Q2.

a) $\tau = I\cdot\overset{\text{acc ang}}{\alpha} \Rightarrow \tau = \frac{dL}{dt} = r\times F =$

$$\mathcal{E} = +\int E\,dl = -\frac{d\phi}{dt} = -\frac{d}{dt}\int B\,da = -\pi b^2\frac{dB}{dt}$$

$$\tau = r\times F = a\lambda E\,\overset{\text{a }dl}{dl} \Rightarrow \tau = a\lambda\int E\,dl = \boxed{-\pi b^2 a\lambda\frac{dB}{dt}}$$

$$L = \int_{B_0}^{0} \tau\,dt = \boxed{+\pi b^2 a\lambda B_0}$$

b) eq. current $\Rightarrow I = 2\pi a\lambda\cdot$ rev freq $= 2\pi a\lambda\cdot\left(\frac{L}{I}\right)$

$$B = \frac{\mu_0}{4\pi}\int\frac{I\,d\vec{l}\times\hat{r}}{r^2} \quad \frac{\mu_0 I}{4\pi}\int\frac{dl}{a^2} = \frac{\mu_0 I}{4\pi a^2}\cdot 2\pi a = \frac{\mu_0 I}{2a}$$

$d\vec{l}\times\hat{r} = dl\cdot\sin 90°$

$$I = \frac{2\pi^2 a^2 b^2\lambda B_0}{I} \Rightarrow \boxed{B = \frac{\mu_0 a\pi^2 b^2\lambda B_0}{I}}$$

$L = I\cdot\omega$

$\omega = \frac{L}{I}$

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300 | 48
288 | 0,6
12

Q3.

a) $E = h\nu \Rightarrow \nu = \frac{c}{\lambda} = \frac{3\cdot 10^{8}}{480\cdot 10^{-9}} = 0{,}6\cdot 10^{15}$

$I = 10\,W/m^2 \Rightarrow P = I\cdot A = 10\,\frac{W}{m^2}\cdot 10^{-4}m^2 = 10^{-3}\,W\,(J/s)$

$E = h\nu = 6{,}63\cdot 10^{-34}\cdot 0{,}6\cdot 10^{15}\,J = 3{,}97\cdot 10^{-19}\,J$

$\text{or} = 4{,}14\cdot 10^{-15}\cdot 0{,}6\cdot 10^{15}\,eV = \boxed{2{,}48\,eV}$

$3{,}6{,}63$
$\cdot 0{,}6$
$3{,}978$

$4{,}14$
$0{,}6$
$2{,}484$

$\frac{10}{2{,}}\ \frac{4}{1{,}2}$

b) $\frac{P}{E} = n = \frac{10^{-3}\,[J/s]}{3{,}97\cdot 10^{-19}\,[J/\text{photon}]} = 1{,}20\cdot 10^{16}\,\text{photons}/s$

c) $20\% \Rightarrow 0{,}24\cdot 10^{16}\,\frac{\text{electrons}}{s} = I$

$K = h\nu - W$

$K_{max} = h\nu - W_0 \rightarrow$ work function

$I = 0{,}24\cdot 10^{16}\cdot e$?

d) $2{,}2 = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{4{,}14\cdot 10^{-15}\cdot 3\cdot 10^{8}}{2{,}2} = 6\cdot 10^{-7} = 600\,nm.$

Q4. $U = \frac{m\omega^2 x^2}{2}$, $\psi(x) = Ae^{-bx^2}$
↳ ground state

a) $-\frac{\hbar^2}{2m}\frac{\partial^2 \psi A e^{-bx^2}}{\partial x^2} + \frac{m\omega^2 x^2}{2}\psi = E A e^{\psi -bx^2}$

$\alpha = \frac{m\omega}{\hbar}$

$y = \sqrt{\alpha} x = \alpha x$

$\frac{m\omega x^2}{\hbar}$

$e^{-\frac{m\omega x^2}{2\hbar}}$

b) $-\frac{\hbar^2}{2m} \cdot A\left(4b^2x^2 \cancel{e^{-bx^2}}\right) + \frac{m\omega^2 x^2}{2}\cancel{\psi} = E A\cancel{e^{-bx^2}}$

$-\frac{\hbar^2}{2m}4b^2x^2 + \frac{m\omega^2 x^2}{2} = E$ $\qquad b^2 = \frac{m^2\omega^2}{4\hbar^2} = \frac{m\omega}{2\hbar}$

$\frac{\hbar^2}{m}2b^2x^2 = \frac{m\omega^2 x^2 - 2E}{2}$

$b = \sqrt{\frac{m^2\omega^2x^2 - 2E}{4\hbar^2x^2}} = \boxed{\frac{m\omega}{2\hbar} - \frac{\sqrt{2E}}{2\hbar x}}$ $\quad -\sqrt{\frac{\hbar\omega}{4\hbar x^2}}$

$\boxed{\text{why } E = 0}$?

$\hat{H}\psi = E\psi$

$E = \left(\frac{1}{2} + n\right)\hbar\omega = \frac{\hbar\omega}{2}$

c) $\int_{-\infty}^{\infty} |\psi(x)|^2 = 1 \Rightarrow A^2 \int_{-\infty}^{\infty} e^{-2bx^2} dx = 1 \Rightarrow A^2 \cdot \left(\frac{\pi}{2b}\right)^{\frac{1}{2}} = 1$

$\boxed{A = \frac{1}{\left(\frac{\pi}{2b}\right)^{\frac{1}{4}}}}$

d) $\int_{-x_{max}}^{x_{max}} |\psi(x)|^2 dx = \int_{-\left(\frac{\hbar}{m\omega}\right)^{\frac{1}{2}}}^{\left(\frac{\hbar}{m\omega}\right)^{\frac{1}{2}}} A^2 e^{-2bx^2} dx$

$\boxed{A = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}}}$

other way to do it!

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Q4.

a) $$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + \frac{1}{2}kx^2\psi(x) = E\,\psi(x)$$

$$\psi'' + \left(-\frac{mkx^2}{\hbar^2} + \frac{2mE}{\hbar^2}\right)\psi = 0 \qquad \alpha^2 = \frac{mk}{\hbar^2}, \quad \lambda = \frac{2mE}{\hbar\alpha}$$

$$\xi = \alpha^{\frac{1}{2}} x$$

$$d\xi = \alpha^{\frac{1}{2}} \Rightarrow \psi''(\xi) + \left(\frac{\alpha\lambda}{\alpha} - \frac{\alpha^2\xi^2}{\alpha}\right)\psi = 0$$

$$\psi''(\xi) - \xi^2\psi(\xi) = 0 \quad \text{for } \xi \to \pm\infty$$

$$\left(\frac{d^2}{d\xi^2} - \xi^2\right)\psi = \left(\frac{d}{d\xi} - \xi\right)\left(\frac{d}{d\xi} + \xi\right)\psi(\xi) = 0$$

$$\psi'(\xi) + \xi\psi(\xi) = 0$$

$$\psi(\xi) = A\,e^{-\xi^2/2} \Rightarrow E = \frac{\lambda\hbar^2\alpha}{2m} = \frac{1}{2}\hbar\sqrt{\frac{k}{m}} = \frac{1}{2}\hbar\omega$$

Q5. $P_A = 32P_0$

a) $U = \frac{3}{2}NRT = \frac{3}{2}PV$, $\frac{S}{N} = \frac{3}{2}R\ln T + R\ln V + C$, isentropic $\Delta S = 0$

$$\Delta S = R\ln\Delta T + R\ln\Delta V \Leftrightarrow \Delta T = \Delta V \Rightarrow NR = \Delta P$$

$NR = 31P_0$ X

$NRT = PV \Rightarrow NR\Delta T = \Delta P\Delta V$

$P_0 = \frac{NR}{31}$

$\Delta S = 0$

$$\frac{3}{2} R \ln \Delta T + R \ln \Delta V = C$$

$$\ln \Delta T^{\frac{3}{2}} + \ln \Delta V = C$$

$$\ln\left(\Delta T^{\frac{3}{2}} \cdot \Delta V\right) = C$$

$$\ln\left(\frac{\Delta P^{\frac{3}{2}} \cdot \Delta V^{\frac{5}{2}}}{(NR)^{\frac{3}{2}}}\right) = C \rightarrow \left(\ln\left(P^3 V^5\right) - 3\ln(NR) = C\right.$$

$$\boxed{P^3 V^5 = \text{costnt}}$$

$$\Delta T = \frac{\Delta P \Delta V}{NR}$$

b) A

$$P_i^3 V_i^5 = P_f^3 V_f^5 \qquad V_i = V_0 \qquad P_f = 32 P_0$$

$$V_f^5 \cdot (32 P_0)^3 = P_0^3 V_0^5$$

$$\boxed{V_f = \frac{V_0}{32^{\frac{3}{5}}}} \rightarrow \Delta V_f = -\frac{V_0}{32^{3/5}} + V_0$$

B

$$V_f = V_0 + \Delta V_f^A$$

$$V_f = -\frac{V_0}{32^{3/5}} + 2V_0$$

c) A

$$NRT_0 = P_0 V_0$$

$$NRT_f = 32 P_0 \cdot \frac{V_0}{32^{3/5}}$$

$$T_f = T_0 \cdot 32^{\frac{2}{5}}$$

B

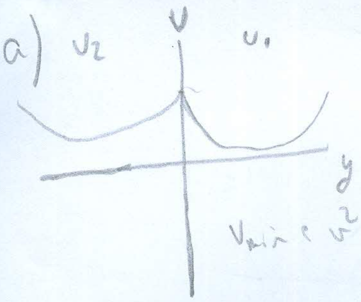
$$NRT_f = P_f V_0 \left(2 - \frac{1}{32^{3/5}}\right)$$

d) $dU = dQ - dW$

EvF 2013, 2Sem $\quad v_{min} \to E$

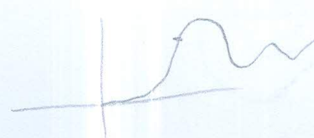
Q6.

a)



$E_1 = \frac{mv_1^2}{2} + U_1 \Rightarrow U_1 = E - \frac{mv_1^2}{2}$

$E_2 = \frac{mv_2^2}{2} + U_2$



b) $E_2 = E_1$

$$v_2 = \sqrt{\left(\frac{mv_1^2}{2} + U_1 - U_2\right)\cdot\frac{2}{m}}$$

$v_{min} \Rightarrow v_2 = 0$

$$\frac{mv_1^2}{2} + U_1 - U_2 = 0 \Rightarrow v_{min} = \sqrt{(U_2 - U_1)\frac{2}{m}}$$

c) $\sin\theta_1 = \frac{x_1}{v_1}$

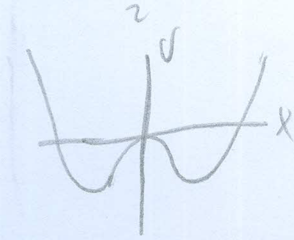
$$\dot{x}^2 = \left(\frac{1}{4}x^4 - cx^2 - E\right)\frac{2}{m}$$

Q7. $U(x) = \frac{1}{2}x^4 - cx^2$

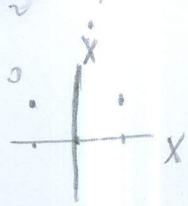
$E = \frac{m\dot{x}^2}{2} + \frac{1}{4}x^4 - cx^2 \Rightarrow$ p/ $E = 0$

$\dot{x} = \sqrt{\frac{x^4}{2m} - \frac{2cx^2}{m}}$

c)

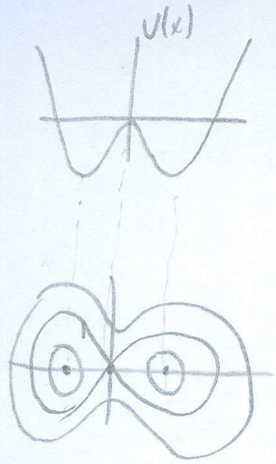


i) $c > 0$

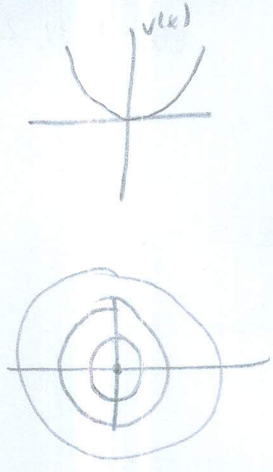


$\theta = \frac{1}{2}\left(\frac{x^4}{2m} - \frac{2cx^2}{m}\right)^{-\frac{1}{2}}$

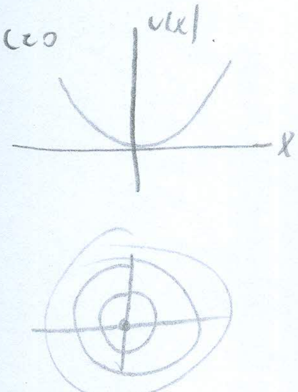
$c>0$



$c=0$



$c<0$



b) $E = \frac{mv^2}{2} + \frac{1}{2}x^4 - cx^2$

soluções periódicas que circulam o ponto fixo 0

c) $E = \frac{mv^2}{2} + \frac{1}{2}x^4 \Rightarrow v^2 = \frac{2E}{m} - \frac{1}{2m}x^4$

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Q8 $\left(\frac{\vec{p}^2}{2m} - a\nabla_p^2\right)\Psi(p,t) = i\frac{\partial}{\partial t}\bar{\Psi}(p,t)$

$\Psi(p,t) = \frac{1}{(2\pi\hbar)^{3/2}}\int d^3r\, e^{-i\vec{p}\cdot\vec{r}/\hbar}\,\Psi(r)$

$\Psi(r) = \frac{1}{(2\pi\hbar)^{3/2}}\int d^3p\, e^{ipr/\hbar}\,\Psi(p)$

$i\frac{d}{dt}\int \frac{1}{C} d^3p\, e^{\frac{ipr}{\hbar}}\,\Psi(p) \Rightarrow \frac{1}{C}\int d^3p\, e^{\frac{ipr}{\hbar}}\left(i\frac{d\Psi(p)}{dt}\right)\frac{\vec{p}^2}{2m}$

$= \frac{1}{C}\int d^3p\, \exp\left(\frac{ipr}{\hbar}\right)\cdot\left[\frac{p^2}{2m} - c\nabla_p^2\right]\Psi(p)$

$U = \frac{p^2}{2m} \Rightarrow du = \frac{p}{m}\,dp \Rightarrow \frac{p^2}{2m}\cdot\frac{\hbar}{ir}\exp\left(\frac{ipr}{\hbar}\right) - \int \frac{p}{m}\frac{\hbar}{ir}\exp\left(\frac{ipr}{\hbar}\right)dp$

$dv = \exp\left(\frac{ipr}{\hbar}\right)$

$u = \frac{p}{m} \quad du = \frac{1}{m}$

$v = \exp\left(\frac{ipr}{\hbar}\right)\cdot\frac{\hbar}{ir}$

$dv = \frac{\hbar}{ir}\exp\left(\frac{ipr}{\hbar}\right) \Rightarrow v = \left(\frac{\hbar}{ir}\right)^2\exp\left(\frac{ipr}{\hbar}\right)$

$\frac{p^2}{2m}\frac{\hbar}{ir}\exp\left(\frac{ipr}{\hbar}\right) + \frac{p}{m}\cdot\left(\frac{\hbar}{ir}\right)^2\exp\left(\frac{ipr}{\hbar}\right)$

Q5

a) $[\hat{S_x}, \hat{S_y}] = \hat{S_x}\hat{S_y} - \hat{S_y}\hat{S_x} = \frac{\hbar^2}{2}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix} - \frac{\hbar^2}{2}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

$= \frac{\hbar^2}{2}\begin{pmatrix} i & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & -i \end{pmatrix} - \frac{\hbar^2}{2}\begin{pmatrix} -i & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & i \end{pmatrix} = \frac{\hbar^2}{2}\begin{pmatrix} 2i & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -2i \end{pmatrix} = i\hbar^2\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

$\boxed{= i\hbar \hat{S_z}}$

$[S_y, S_z] = 2i\hbar S_x$ ... permutation cyclic]

b) $\hat{S_z}|\psi\rangle = \hbar \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} |\psi\rangle \begin{pmatrix} 1 \\ i \\ 2 \end{pmatrix} = \hbar \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ $\quad \begin{bmatrix} h \\ 0 \\ 0 \end{bmatrix}$

possíveis valores $\Rightarrow h, -h$ (auto valores) $\quad \hat{\ } = h \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\begin{bmatrix} h & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -h \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = h\begin{bmatrix} a \\ b \\ c \end{bmatrix} \Rightarrow \begin{matrix} ah = ah \\ 0 = hb \\ -hc = hc \end{matrix} \rightarrow \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$\lambda = -h$

$\begin{matrix} ah = -ha \\ 0 = -hb \\ -hc = -hc \end{matrix} \Rightarrow \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$; $\begin{matrix} ah = 0 \\ 0 = 0 \\ hc = 0 \end{matrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $\Rightarrow \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$P\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \frac{1}{6}$; $P\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \frac{4}{6} = \frac{2}{3}$

c) $|\phi\rangle = \begin{pmatrix} 1 \\ i \\ -2 \end{pmatrix} = 1|a\rangle + i|b\rangle + -2|c\rangle$

$A^2(1+1+4) = 1 \Rightarrow \frac{1}{\sqrt{6}}\left[|a\rangle + i|b\rangle - 2|c\rangle\right]^2 \sqrt{d|}$?

$A = \sqrt{\frac{1}{6}}$

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Q10.

$kx^2, x>0$

a) 

$\varepsilon_n = (n+\frac{1}{2})\hbar\omega \quad n = 1, 3, 5, \ldots$

ψ_n é par p/ n par

ψ_n é ímpar p/ n ímpar

$\hat{p}\,\psi(x) = \psi(-x) = 0$

b) $z = \sum e^{-\beta E_n} = e^{-\beta\frac{1}{2}\hbar\omega} \cdot \sum e^{-\beta n\hbar\omega} = \sum \left(e^{-\beta\hbar\omega}\right)^n = e$